Access to Science, Engineering and Agriculture: Mathematics 1 MATH00030 Chapter 6 Solutions

1. (a)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{4(x+h)-4x}{h} \\ &= \lim_{h\to 0} \frac{4x+4h-4x}{h} \\ &= \lim_{h\to 0} \frac{4h}{h} \\ &= \lim_{h\to 0} 4 \\ &= 4. \end{aligned}$$

(b)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{5(x+h)^2-5x^2}{h} \\ &= \lim_{h\to 0} \frac{5(x^2+2xh+h^2)-5x^2}{h} \\ &= \lim_{h\to 0} \frac{5x^2+10xh+5h^2-5x^2}{h} \\ &= \lim_{h\to 0} \frac{10xh+5h^2}{h} \\ &= \lim_{h\to 0} 10x+5h \\ &= 10x. \end{aligned}$$

(c)

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{-3(x+h)+2-(-3x+2)}{h} \\ &= \lim_{h\to 0} \frac{-3x-3h+2+3x-2}{h} \\ &= \lim_{h\to 0} \frac{-3h}{h} \\ &= \lim_{h\to 0} -3 \\ &= -3. \end{aligned}$$

(d)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{4(x+h)^2-5-(4x^2-5)}{h} \\
&= \lim_{h\to 0} \frac{4(x^2+2xh+h^2)-5-(4x^2-5)}{h} \\
&= \lim_{h\to 0} \frac{4x^2+8xh+4h^2-5-4x^2+5}{h} \\
&= \lim_{h\to 0} \frac{8xh+4h^2}{h} \\
&= \lim_{h\to 0} 8x+4h \\
&= 8x.
\end{aligned}$$

(e)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{-(x+h)^2+2(x+h)+3-(-x^2+2x+3)}{h} \\
&= \lim_{h\to 0} \frac{-(x^2+2xh+h^2)+2(x+h)+3-(-x^2+2x+3)}{h} \\
&= \lim_{h\to 0} \frac{-x^2-2xh-h^2+2x+2h+3+x^2-2x-3}{h} \\
&= \lim_{h\to 0} \frac{-2xh-h^2+2h}{h} \\
&= \lim_{h\to 0} -2x-h+2 \\
&= -2x+2.
\end{aligned}$$

2. (a) Since $f(x) = 5$ is a constant, $f'(x) = 0$.
   (b) Since $f(x) = -\pi\cos(e)$ is a constant, $f'(x) = 0$.
   (c) Here f is of the form $f(x) = x^n$, with $n = 2$.
   Thus $f'(x) = 2x^{2-1} = 2x^1 = 2x$.
   (d) Here f is of the form $f(x) = x^n$, with $n = \frac{9}{2}$.
   Thus $f'(x) = \frac{9}{2}x^{\frac{9}{2}-1} = \frac{9}{2}x^{\frac{7}{2}}$.
   (e) Here f is of the form $f(x) = x^n$, with $n = -5$.
   Thus $f'(x) = -5x^{-5-1} = -5x^{-6}$.
   (f) Here f is of the form $f(x) = x^n$, with $n = \cos(2)$.
   Thus $f'(x) = \cos(2)x^{\cos(2)-1}$.
   (g) Here f is of the form $f(x) = e^{ax}$, with $a = 4$.
   Thus $f'(x) = 4e^{4x}$.

(h) Here f is of the form $f(x) = e^{ax}$, with $a = \frac{3}{2}$.
Thus $f'(x) = \frac{3}{2} e^{\frac{3}{2}x}$.

(i) Here f is of the form $f(x) = e^{ax}$, with $a = -6$.
Thus $f'(x) = -6e^{-6x}$.

(j) Here f is of the form $f(x) = e^{ax}$, with $a = \pi$.
Thus $f'(x) = \pi e^{\pi x}$.

(k) Here f is of the form $f(x) = \ln(ax)$, with $a = 4$.
Thus $f'(x) = \frac{1}{x}$.

(l) Here f is of the form $f(x) = \ln(ax)$, with $a = -\pi$.
Thus $f'(x) = \frac{1}{x}$.

(m) Here f is of the form $f(x) = \ln(ax)$, with $a = \frac{1}{2}$.
Thus $f'(x) = \frac{1}{x}$.

(n) Here f is of the form $f(x) = \sin(ax)$, with $a = 2$.
Thus $f'(x) = 2\cos(2x)$.

(o) Here f is of the form $f(x) = \sin(ax)$, with $a = -2$.
Thus $f'(x) = -2\cos(-2x)$.

(p) Here f is of the form $f(x) = \sin(ax)$, with $a = e$.
Thus $f'(x) = e\cos(ex)$.

(q) Here f is of the form $f(x) = \cos(ax)$, with $a = 3$.
Thus $f'(x) = -3\sin(3x)$.

(r) Here f is of the form $f(x) = \cos(ax)$, with $a = -3$.
Thus $f'(x) = -(-3)\sin(-3x) = 3\sin(-3x)$.

(s) Here f is of the form $f(x) = \cos(ax)$, with $a = -\pi$.
Thus $f'(x) = -(-\pi)\sin(-\pi x) = \pi\sin(-\pi x)$.

3. (a) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}(1) + \frac{d}{dx}(3x) + \frac{d}{dx}\left(-2x^2\right) + \frac{d}{dx}\left(3x^3\right) + \frac{d}{dx}\left(-4x^4\right) \\ &= \frac{d}{dx}(1) + 3\frac{d}{dx}(x) - 2\frac{d}{dx}\left(x^2\right) + 3\frac{d}{dx}\left(x^3\right) - 4\frac{d}{dx}\left(x^4\right) \\ &= 0 + 3(1) - 2(2x) + 3(3x^2) - 4(4x^3) \\ &= 3 - 4x + 9x^2 - 16x^3. \end{aligned}$$

Note that in your assignment or exam solutions you don't need to give as much detail as this. I am just setting out everything carefully until you get used to the ideas involved.

(b) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(-x^{-1}\right) + \frac{d}{dx}\left(2\sin 4x\right) \\ &= -\frac{d}{dx}\left(x^{-1}\right) + 2\frac{d}{dx}\left(\sin 4x\right) \\ &= -\left(-x^{-2}\right) + 2(4\cos(4x)) \\ &= x^{-2} + 8\cos(4x). \end{aligned}$$

(c) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(3e^{-\frac{1}{2}x}\right) + \frac{d}{dx}\left(-2\cos\left(\frac{1}{2}x\right)\right) \\ &= 3\frac{d}{dx}\left(e^{-\frac{1}{2}x}\right) - 2\frac{d}{dx}\left(\cos\left(\frac{1}{2}x\right)\right) \\ &= 3\left(-\frac{1}{2}e^{-\frac{1}{2}x}\right) - 2\left(-\frac{1}{2}\sin\left(\frac{1}{2}x\right)\right) \\ &= -\frac{3}{2}e^{-\frac{1}{2}x} + \sin\left(\frac{1}{2}x\right). \end{aligned}$$

(d) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(2\ln(-x)\right) + \frac{d}{dx}\left(4\cos(-3x)\right) + \frac{d}{dx}\left(-e^{-\frac{3}{2}x}\right) \\ &= 2\frac{d}{dx}\left(\ln(-x)\right) + 4\frac{d}{dx}\left(\cos(-3x)\right) - \frac{d}{dx}\left(e^{-\frac{3}{2}x}\right) \\ &= 2\left(\frac{1}{x}\right) + 4\left(3\sin(-3x)\right) - \left(-\frac{3}{2}e^{-\frac{3}{2}x}\right) \\ &= \frac{2}{x} + 12\sin(-3x) + \frac{3}{2}e^{-\frac{3}{2}x}. \end{aligned}$$

(e) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(-2x^2\right) + \frac{d}{dx}\left(3\ln(3x)\right) + \frac{d}{dx}\left(e^{\cos(1)x}\right) \\ &= -2\frac{d}{dx}\left(x^2\right) + 3\frac{d}{dx}\left(\ln(3x)\right) + \frac{d}{dx}\left(e^{\cos(1)x}\right) \\ &= -2\left(2x\right) + 3\left(\frac{1}{x}\right) + \cos(1)e^{\cos(1)x} \\ &= -4x + \frac{3}{x} + \cos(1)e^{\cos(1)x}. \end{aligned}$$

(f) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(2\sin(3x)\right) + \frac{d}{dx}\left(-3\sin(2x)\right) + \frac{d}{dx}\left(2\cos(3x)\right) + \frac{d}{dx}\left(-3\cos(2x)\right) \\ &= 2\frac{d}{dx}\left(\sin(3x)\right) - 3\frac{d}{dx}\left(\sin(2x)\right) + 2\frac{d}{dx}\left(\cos(3x)\right) - 3\frac{d}{dx}\left(\cos(2x)\right) \\ &= 2\left(3\cos(3x)\right) - 3\left(2\cos(2x)\right) + 2\left(-3\sin(3x)\right) - 3\left(-2\sin(2x)\right) \\ &= 6\cos(3x) - 6\cos(2x) - 6\sin(3x) + 6\sin(2x) \\ &= 6(\cos(3x) - \cos(2x) - \sin(3x) + \sin(2x)). \end{aligned}$$

(g) Using the sum rule,

$$f'(x) = \frac{d}{dx}\left(e^2 - 4\right) + \frac{d}{dx}\left(e^{2x}\right) = 0 + 2e^{2x} = 2e^{2x}.$$

Note that here we didn't need the multiple rule and also we were able to deal with the two terms e^2 and -4 all at once since $e^2 - 4$ is just a constant.

(h) Using the sum and multiple rules,

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(-3x^{-3}\right) + \frac{d}{dx}\left(4x^4\right) + \frac{d}{dx}\left(5x^{-5}\right) + \frac{d}{dx}(3) \\ &= -3\frac{d}{dx}\left(x^{-3}\right) + 4\frac{d}{dx}\left(x^4\right) + 5\frac{d}{dx}\left(x^{-5}\right) + \frac{d}{dx}(3) \\ &= -3\left(-3x^{-4}\right) + 4\left(4x^3\right) + 5\left(-5x^{-6}\right) + 0 \\ &= 9x^{-4} + 16x^3 - 25x^{-6}. \end{aligned}$$

Note that $3x^0$ is just the number 3 (unless $x = 0$ when x^0 is not defined), so it differentiates to zero. We could also obtain the derivative as $3\left(0x^{-1}\right) = 0$ but it would be a bit more work.